

# $J^P = \frac{1}{2}^-$ Pentaquarks in Jaffe and Wilczek's Diquark Model

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If Jaffe and Wilczek's diquark picture for  $\Theta_5^+$  pentaquark is correct, there should also exist a  $SU_F(3)$  pentaquark octet and singlet with no orbital excitation between the diquark pair, hence  $J^P = \frac{1}{2}^-$ . These states are lighter than the  $\Theta_5^+$  anti-decuplet and lie close to the orbitally excited ( $L=1$ ) three-quark states in the conventional quark model. We calculate their masses and magnetic moments and discuss their possible strong decays using the chiral Lagrangian formalism. Among them two pentaquarks with nucleon quantum numbers may be narrow. Selection rules of strong decays are derived. We propose the experimental search of these nine additional  $J^P = \frac{1}{2}^-$  baryon states. Especially there are two additional  $J^P = \frac{1}{2}^-$   $\Lambda$  baryons around  $\Lambda(1405)$ . We also discuss the interesting possibility of interpreting  $\Lambda(1405)$  as a pentaquark. The presence of these additional states will provide strong support of the diquark picture for the pentaquarks. If future experimental searches fail, one has to re-evaluate the relevance of this picture for the pentaquarks.

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## I. INTRODUCTION

Since LEPS announced the surprising discovery of the very narrow  $\Theta_5^+$  pentaquark ( $uudd\bar{s}$ ) around 1540 MeV last year [1], many other experimental groups have claimed the observation of evidence of its existence [2, 3, 4, 5, 6, 7, 8, 9] while a few groups reported negative results [10, 11]. Preliminary experimental data indicate that  $\Theta_5^+$  is an iso-scalar. Later, NA49 Collaboration [12] reported a second narrow pentaquark  $\Xi_5^{--}$  ( $ddss\bar{u}$ ) at 1862 MeV, to which serious challenge is raised in Ref. [13]. Very recently, H1 Collaboration reported the discovery of the anti-charmed pentaquark [14].

One can use some textbook group theory to write down the wave functions in the framework of quark model. Because of its low mass, high orbital excitation with  $L \geq 2$  is unlikely. Pauli principle requires the totally anti-symmetric wave functions for the four light quarks. Since the anti-quark is in the  $[11]_C$  representation, the four quark color wave function is  $[211]_C$ .

With  $L = 0$ , hence  $P = -$ , the 4q spatial wave function is symmetric, i.e.,  $[4]_O$ . Their  $SU(6)_{FS}$  spin-flavor wave function must be  $[31]_{FS}^{210}$  which contains  $[22]_F^6 \times [31]_S^3$  after decomposition into  $SU(3)_F \times SU(2)_S$  [15, 16, 17]. Here 210, 6 etc is the dimension of the representation. When combined with anti-quark, we get  $([33]_F^{10} + [21]_F^8) \times ([41]_S + [32]_S)$ , which is nothing but  $(10_F + 8_F) \times ((\frac{3}{2})_S + (\frac{1}{2})_S)$  in terms of more common notation. The total angular momentum of the four quarks is one. The resulting exotic anti-decuplet is always ac-

companied by a nearly degenerate octet. Their angular momentum and parity is either  $J^P = \frac{3}{2}^-$  or  $\frac{1}{2}^-$ .

With  $L = 1$  and  $P = +$ , the four quark  $SU(6)_{OFS}$  space-spin-flavor wave function must be  $[31]_{OFS}$ . Only this representation can combine with  $[211]_C$  color wave function to ensure that the 4q total wave functions are anti-symmetric. The 4q orbital wave function is  $[31]_O$ . There are several  $SU(6)_{FS}$  wave functions  $[4]_{FS}, [31]_{FS}, [22]_{FS}, [211]_{FS}$  which allow the 4q total wave function to be anti-symmetric and lead to the octet and exotic anti-decuplet [16, 17].

At present there are two outstanding pending issues: the  $\Theta_5^+$  parity and its narrow width. Two earlier lattice QCD simulation favored negative parity for  $\Theta_5^+$  [18] while a recent one advocates positive parity [19]. Theoretical papers can be roughly classified into two categories according to  $\Theta_5^+$  parity.

The parity of the anti-decuplet is positive in the chiral soliton model [20, 21, 22]. But the foundation of this framework is questioned in Ref. [23, 24]. Several clustered quark models were constructed to let the anti-decuplet carry positive parity [25, 26, 27]. There are other models favoring positive parity [28, 29, 30].

On the other hand, QCD sum rule approach favors negative parity for  $\Theta_5^+$  [31, 32]. Recently in the framework of the flux tube model  $\Theta_5^+$  pentaquark is proposed to have a extremely stable diamond structure with negative parity [33]. There are also many models supporting negative parity [34, 35, 36]. Many schemes have been proposed to determine the pentaquark parity experimentally [37].

All experiments indicate the  $\Theta_5^+$  pentaquark is very narrow. More stringent constraint on its decay width comes from the reanalysis of the previous kaon nucleon scattering data, which sets an upper bound of one or

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two MeV [38]. Otherwise, the  $\Theta_5^+$  pentaquark should not have escaped detection. The width of a conventional baryon 100 MeV above the threshold is typically 100 MeV or bigger. Therefore, the extremely narrow width is very puzzling.

Recently there appeared several interesting schemes for the narrow width. Within the chiral soliton model, the coupling constants in the leading order, next-leading order and next-next-leading order large  $N_c$  expansion cancel almost completely, which leads to a narrow width [22]. It is suggested that one of two nearly degenerate pentaquarks can be arranged to decouple from the decay modes after diagonalizing the mixing mass matrix via kaon nucleon loop [39]. After constructing a special pentaquark wave function with the color-orbital part being totally anti-symmetric, the overlap amplitude between the final state and pentaquark is suppressed significantly [35], which may also explain the narrow width. Such a scheme is based on the mismatch of initial and final state spin-flavor wave functions [17, 40, 41]. With the stable diamond structure the system undergoes a special structural phase transition when the  $\Theta_5^+$  pentaquark decays into the planar kaon and nucleon. The non-planar flux tubes were broken and new planar ones are formed. Hence the decay width of the  $\Theta_5^+$  pentaquark should be small [33].

In this paper we will study the phenomenology of Jaffe and Wilczek's diquark model for the pentaquarks. We note the problem with the identification of the ideally mixed positive parity pentaquarks with the  $N(1710)$  and  $N(1440)$  is discussed extensively in [42]. If the diquark model is correct, there should exist a  $SU_F(3)$  pentaquark octet and singlet with no orbital excitation between the diquark pair and  $J^P = \frac{1}{2}^-$ . These states are lighter than the  $\Theta_5^+$  anti-decuplet and lie close to the orbitally excited ( $L=1$ ) three-quark states in the conventional quark model.

Our paper is organized as follows: Section I is a brief review of the field. In Section II we use JW's model [25] to calculate the masses and magnetic moments of the pentaquark octet and singlet which arise from  $\mathbf{3}_F \otimes \mathbf{3}_F$ , where the diquark-diquark system is in the flavor  $\mathbf{3}_F$  and the antiquark is in the flavor  $\mathbf{3}_F$ . Our present pentaquark octet is in the mixed antisymmetric representation, which is different from the mixed symmetric pentaquark octet accompanying the anti-decuplet. In Section III we discuss strong decays of these states. In Section IV we derive selection rules in the case of ideal mixing. The final section is a short summary.

## II. MASSES AND MAGNETIC MOMENTS OF THE PENTAQUARK OCTET AND SINGLET

Jaffe and Wilczek [25] proposed that pentaquark states are composed of two scalar diquarks and one anti-quark. Diquarks obey Bose statistics. Each diquark is in the antisymmetric color  $\mathbf{3}$  state. The spin wave function of the

two quarks within each scalar diquark is antisymmetric while the spatial part is symmetric. Pauli principle requires the total wave function of the two quarks in the diquark be anti-symmetric. Thus the flavor wave function of the two quarks in the diquark must be antisymmetric, i.e., the diquark is in the flavor  $\mathbf{\bar{3}}_F$  state. The diquark and antiquark flavor wave functions are listed in Table I.

$(Y, I, I_3)$	Flavor wave functions
$(\frac{2}{3}, 0, 0)$	$[ud], \bar{s}$
$(-\frac{1}{3}, \frac{1}{2}, \frac{1}{2})$	$[su], \bar{d}$
$(-\frac{1}{3}, \frac{1}{2}, -\frac{1}{2})$	$[ds], \bar{u}$

TABLE I: Diquark and antiquark flavor wave functions. Where  $Y, I$  and  $I_3$  are hypercharge, isospin and the third component of isospin respectively.  $[q_1 q_2] = \frac{1}{\sqrt{2}}(q_1 q_2 - q_2 q_1)$ .

The color wave function of the two diquarks within the pentaquark must be antisymmetric  $\mathbf{3}_C$ . In order to get an exotic anti-decuplet, the two scalar diquarks combine into the symmetric  $SU(3)$   $\mathbf{6}_F$ :  $[ud]^2$ ,  $[ud][ds]_+$ ,  $[su]^2$ ,  $[su][ds]_+$ ,  $[ds]^2$ , and  $[ds][ud]_+$ . Bose statistics demands symmetric total wave function of the diquark-diquark system, which leads to the antisymmetric spatial wave function with one orbital excitation. The resulting anti-decuplet and octet pentaquarks have  $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ .

We note that lighter pentaquarks can be formed if the two scalar diquarks are in the antisymmetric  $SU(3)_F$   $\mathbf{3}$  representation:  $[ud][su]_-$ ,  $[ud][ds]_-$ , and  $[su][ds]_-$ , where  $[q_1 q_2][q_3 q_4]_- = \sqrt{\frac{1}{2}}([q_1 q_2][q_3 q_4] - [q_3 q_4][q_1 q_2])$ . No orbital excitation is needed to ensure the symmetric total wave function of two diquarks since the spin-flavor-color part is symmetric. The total angular momentum of these pentaquarks is  $\frac{1}{2}$  and the parity is negative. There is no accompanying  $J = \frac{3}{2}$  multiplet. The two diquarks combine with the antiquark to form a  $SU(3)_F$  octet and singlet pentaquark multiplet:  $\mathbf{3}_F \otimes \mathbf{3}_F = \mathbf{8}_F \oplus \mathbf{1}_F$ . The flavor wave functions of the pentaquarks are listed in Table II. Similar mechanism has been proposed to study heavy pentaquarks with negative parity and lighter mass than  $\Theta_{c,b}$  in [43, 44].

According to JW's model [25], the strange quark mass explicitly breaks  $SU(3)_F$  symmetry. The  $[ud]$  diquark is more tightly bound than  $[us]$  and  $[ds]$ . The energy difference can be related to the  $\Sigma$ - $\Lambda$  mass splitting. Thus, every strange quark in the pentaquark contributes  $\alpha \equiv \frac{3}{4}(M_\Sigma - M_\Lambda) \approx 60$  MeV arising from  $[ud]$  and  $[us]$ ,  $[ds]$  binding energy difference. The Hamiltonian in JW's model reads

$$H_s = M_0 + (n_s + n_{\bar{s}})m_s + n_s \alpha \quad (1)$$

where  $M_0$  is the pentaquark mass in the  $SU(3)_F$  symmetry limit. The last two terms are from  $SU(3)_F$  symmetry breaking with  $m_s \approx 100$  MeV.  $M_0$  has the form

$$M_0 = 2m_{di} + m_{\bar{q}} + \delta M_l \quad (2)$$

	(Y, I)	$I_3$	Flavor wave functions	Masses (MeV)
$p_8$	$(1, \frac{1}{2})$	$\frac{1}{2}$	$[su][ud]_{-\bar{s}}$	1460
$n_8$		$-\frac{1}{2}$	$[ds][ud]_{-\bar{s}}$	1460
$\Sigma_8^+$	$(0, 1)$	1	$[su][ud]_{-\bar{d}}$	1360
$\Sigma_8^0$		0	$\frac{1}{\sqrt{2}}([su][ud]_{-\bar{u}} + [ds][ud]_{-\bar{d}})$	1360
$\Sigma_8^-$		-1	$[ds][ud]_{-\bar{u}}$	1360
$\Lambda_8$	$(0, 0)$	0	$\frac{[ud][su]_{-\bar{u}} + [ds][ud]_{-\bar{d}} - 2[su][ds]_{-\bar{s}}}{\sqrt{6}}$	1533
$\Xi_8^0$	$(-1, \frac{1}{2})$	$\frac{1}{2}$	$[ds][su]_{-\bar{d}}$	1520
$\Xi_8^-$		$-\frac{1}{2}$	$[ds][su]_{-\bar{u}}$	1520
$\Lambda_1$	$(0, 0)$	0	$\frac{[ud][su]_{-\bar{u}} + [ds][ud]_{-\bar{d}} + [su][ds]_{-\bar{s}}}{\sqrt{3}}$	1447

TABLE II: Flavor wave functions and masses of the  $\frac{1}{2}^-$  pentaquark octet and singlet.

where  $m_{di}$  is the  $[ud]$  diquark mass,  $m_{\bar{q}}$  is the anti-quark mass and  $\delta M_l$  is the orbital excitation energy. We follow Ref. [25] to use  $m_{di} = 420$  MeV,  $m_{\bar{q}} = 360$  MeV and  $\delta M_l = 0$  for  $l = 0$  to get  $M_0 = 1200$  MeV. Thus we can use Eq.(1) to compute masses of the pentaquark octet and singlet. The numerical results are collected in Table II.

There are three mass relations among the nine pentaquarks. First we get the Gell-Mann-Okubo relation for the pentaquark octet

$$2M_{N_8} + 2M_{\Xi_8} = 3M_{\Lambda_8} + M_{\Sigma_8}, \quad (3)$$

which is similar to that for the ground state octet. We also have

$$\begin{aligned} M_{\Lambda_8} - M_{\Lambda_1} &= M_{\Lambda_1} - M_{\Sigma_8} \\ M_{\Lambda_8} - M_{N_8} &= M_{\Xi_8} - M_{\Lambda_1}. \end{aligned} \quad (4)$$

The pentaquark magnetic moment has the form [45]

$$\vec{\mu} = \sum_i \vec{\mu}_i = \sum_i (g_i \vec{s}_i + \vec{l}_i) \mu_i, \quad (5)$$

where  $\vec{s}_i$ ,  $\vec{l}_i$  are the spin and orbital momentum of the  $i$ -th constituent respectively.  $g_i$  is the g-factor of the  $i$ -th constituent and  $\mu_i$  is the magneton of the  $i$ -th constituent. The spin of the scalar diquark is zero. There is no orbital momentum. So the magnetic moment of  $J^P = \frac{1}{2}^-$  pentaquark  $\vec{\mu}$  simply reads

$$\begin{aligned} \vec{\mu} &= (g_1 \vec{0} + \vec{0})\mu_1 + (g_2 \vec{0} + \vec{0})\mu_2 + (g_3 \frac{1}{2} \vec{1} + \vec{0})\mu_3 \\ &= g_3 \frac{1}{2} \mu_3, \end{aligned} \quad (6)$$

where 1,2 denote the two scalar diquarks and 3 denotes the anti-quark. It is clear that the pentaquark magnetic moment arises from the anti-quark only. Finally we get

$$\mu = \mu_{\bar{q}} = \frac{e_{\bar{q}}}{2m_{\bar{q}}} = -\frac{e_q}{2m_q} \quad (7)$$

where  $e_{\bar{q}}$  is the charge of the antiquark and  $m_{\bar{q}}$  is the mass of the antiquark. We present the expressions and numerical results of octet and singlet pentaquark magnetic moments in Table III.

	(Y, I)	$I_3$	Magnetic moments	Numerical results ( $\mu_N$ )
$p_8$	$(1, \frac{1}{2})$	$\frac{1}{2}$	$\frac{e_0}{6m_s}$	0.63
$n_8$		$-\frac{1}{2}$	$\frac{e_0}{6m_s}$	0.63
$\Sigma_8^+$	$(0, 1)$	1	$\frac{e_0}{6m_d}$	0.87
$\Sigma_8^0$		0	$\frac{1}{6}(-\frac{e_0}{m_u} + \frac{e_0}{2m_d})$	-0.43
$\Sigma_8^-$		-1	$-\frac{e_0}{3m_u}$	-1.74
$\Lambda_8$	$(0, 0)$	0	$\frac{1}{18}(-\frac{e_0}{m_u} + \frac{e_0}{2m_d} + \frac{2e_0}{m_s})$	0.27
$\Xi_8^0$	$(-1, \frac{1}{2})$	$\frac{1}{2}$	$\frac{e_0}{6m_d}$	0.87
$\Xi_8^-$		$-\frac{1}{2}$	$-\frac{e_0}{3m_u}$	-1.74
$\Lambda_1$	$(0, 0)$	0	$\frac{1}{9}(-\frac{e_0}{m_u} + \frac{e_0}{2m_d} + \frac{e_0}{2m_s})$	-0.08

TABLE III: Expressions and numerical results of the magnetic moments of the pentaquark octet and singlet, where  $e_0$  is the charge unit.

There exist several magnetic moment relations.

$$\begin{aligned} \mu_{\Lambda_8} - \mu_{\Sigma_8^0} &= 2(\mu_{n_8} - \mu_{\Lambda_8}) = 2(\mu_{\Lambda_8} - \mu_{\Lambda_1}) \\ \mu_{\Sigma_8^+} - \mu_{\Sigma_8^0} &= \mu_{\Sigma_8^0} - \mu_{\Sigma_8^-} \\ \mu_{n_8} - \mu_{\Lambda_1} &= 2(\mu_{\Lambda_1} - \mu_{\Sigma_8^0}) \\ \mu_{p_8} &= \mu_{n_8} \\ \mu_{\Xi_8^0} &= \mu_{\Sigma_8^+} \\ \mu_{\Xi_8^-} &= \mu_{\Sigma_8^-}. \end{aligned} \quad (8)$$

We note only the second one is similar to the Coleman-Glashow relations for nucleon octet [46].

### III. PENTAQUARK CHIRAL LAGRANGIAN, STRONG DECAYS AND SELECTION RULES

In the case of pentaquark decays, if symmetry and kinematics allow, the most efficient decay mechanism is for the four quarks and anti-quark to regroup with each other into a three-quark baryon and a meson. This is in contrast to the  $^3P_0$  decay models for the ordinary hadrons. This regrouping is coined as the "fall-apart" mechanism in Refs. [17, 35, 40, 41].

In the following we write down the interaction chiral Lagrangian using  $SU(3)_F$  symmetry. We denote a quark and anti-quark by  $q^i, \bar{q}_j$  where  $i, j$  are the  $SU(3)_F$  flavor indices. Note that the flavor wave function of the  $J^P = \frac{1}{2}^-$  octet and singlet pentaquark arise from

$$A_{[ij]} \otimes \bar{q}_k = S \oplus O_{[ij,k]}, \quad (9)$$

where the indices  $ij$  are antisymmetric,  $A_{[ij]}$  is the  $\mathbf{3}_F$  diquark pair.  $S$  is the pentaquark singlet whose indices are contracted completely.  $O_{[ij,k]}$  is the octet representation.

The index  $k$  represents the antiquark which contracts with one of the meson index.

In Ref. [47] the chiral Lagrangian is built to discuss the decay modes of the anti-decuplet and octet with positive parity. The authors pointed out that keeping explicit track of the flavor indices of the two diquarks minimize the independent coupling constants and lead to some selection rules.

For the interaction of the  $J^P = \frac{1}{2}^-$  pentaquark octet  $P$ , nucleon octet  $B$  and pseudoscalar meson octet  $M$ , we have

$$\mathcal{L}_8 = g_8 \epsilon_{ilm} \bar{O}^{[ij,k]} B_j^l M_k^m + H.c., \quad (10)$$

where  $O_{[ij,k]} = \epsilon_{ljk} P_i^l - \epsilon_{lik} P_j^l$ . The explicit form of the matrix  $B_j^i$ ,  $M_j^i$  and  $P_j^i$  is

$$(P_j^i) = \begin{pmatrix} \frac{\Sigma_8^0}{\sqrt{2}} + \frac{\Lambda_8}{\sqrt{6}} & \Sigma_8^+ & p_8 \\ \Sigma_8^- & -\frac{\Sigma_8^0}{\sqrt{2}} + \frac{\Lambda_8}{\sqrt{6}} & n_8 \\ \Xi_8^- & \Xi_8^0 & -\frac{2\Lambda_8}{\sqrt{6}} \end{pmatrix}, \quad (11)$$

$$(B_j^i) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}, \quad (12)$$

$$(M_j^i) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_0}{\sqrt{6}} \end{pmatrix}. \quad (13)$$

We present the Clebsch-Gordan coefficient of each interaction term in Table IV.

$\Xi_8^-$	$\Xi_8^0$	$p_8$	$n_8$
$\Xi^- \pi^0$ $\frac{1}{\sqrt{2}}$	$\Xi^- \pi^+$ 1	$\Sigma^0 K^+$ $\frac{1}{\sqrt{2}}$	$\Sigma^- K^+$ 1
$\Xi^0 \pi^-$ 1	$\Xi^0 \pi^0$ $-\frac{1}{\sqrt{2}}$	$\Sigma^+ K^0$ 1	$\Sigma^0 K^0$ $-\frac{1}{\sqrt{2}}$
$\Xi^- \eta_0$ $\frac{1}{\sqrt{6}}$	$\Xi^0 \eta_0$ $\frac{1}{\sqrt{6}}$	$p \eta_0$ $-\frac{2}{\sqrt{6}}$	$n \eta_0$ $-\frac{2}{\sqrt{6}}$
$\Lambda K^-$ $-\frac{2}{\sqrt{6}}$	$\Lambda \bar{K}^0$ $-\frac{2}{\sqrt{6}}$	$\Lambda K^+$ $\frac{1}{\sqrt{6}}$	$\Lambda K^+$ $\frac{1}{\sqrt{6}}$
$\Sigma_8^0$	$\Sigma_8^+$	$\Sigma_8^-$	$\Lambda_8$
$\Sigma^+ \pi^-$ $\frac{1}{\sqrt{2}}$	$\Sigma^+ \pi^0$ $-\frac{1}{\sqrt{2}}$	$\Sigma^- \pi^0$ $\frac{1}{\sqrt{2}}$	$\Sigma^+ \pi^-$ $\frac{1}{\sqrt{6}}$
$\Sigma^- \pi^+$ $-\frac{1}{\sqrt{2}}$	$\Sigma^0 \pi^+$ $\frac{1}{\sqrt{2}}$	$\Sigma^0 \pi^-$ $-\frac{1}{\sqrt{2}}$	$\Sigma^- \pi^+$ $\frac{1}{\sqrt{6}}$
$\Sigma^0 \eta_0$ $\frac{1}{\sqrt{6}}$	$\Sigma^+ \eta_0$ $\frac{1}{\sqrt{6}}$	$\Sigma^- \eta_0$ $\frac{1}{\sqrt{6}}$	$\Sigma^0 \pi^0$ $\frac{1}{\sqrt{6}}$
$p K^-$ $\frac{1}{\sqrt{2}}$	$p \bar{K}^0$ 1	$n K^-$ 1	$p K^-$ $\frac{1}{\sqrt{6}}$
$n \bar{K}^0$ $-\frac{1}{\sqrt{2}}$	$\Lambda \pi^+$ $\frac{1}{\sqrt{6}}$	$\Lambda \pi^-$ $\frac{1}{\sqrt{6}}$	$n \bar{K}^0$ $\frac{1}{\sqrt{6}}$
$\Lambda \pi^+$ $\frac{1}{\sqrt{6}}$			$\Xi^- K^+$ $-\frac{2}{\sqrt{6}}$
			$\Xi^0 K^0$ $-\frac{2}{\sqrt{6}}$
			$\Lambda \eta_0$ $-\frac{1}{\sqrt{6}}$

TABLE IV: Coupling of the  $J^P = \frac{1}{2}^-$  pentaquark octet with the usual baryon octet and the pseudoscalar meson octet. The universal coupling constant  $g_8$  is omitted.

The pentaquark octet can also couple with usual

baryon octet and meson singlet  $\eta_1$ .

$$\begin{aligned} \mathcal{L}_1 &= g_1 \bar{P}_i^j B_j^i \eta_1 + H.c. \\ &= g_1 (\bar{N}_8 N + \bar{\Sigma}_8 \Sigma + \bar{\Xi}_8 \Xi + \bar{\Lambda}_8 \Lambda) \eta_1 + H.c., \end{aligned} \quad (14)$$

where

$$N = \begin{pmatrix} p \\ n \end{pmatrix}, \quad \Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}. \quad (15)$$

The interaction among pentaquark singlet  $\Lambda_1$ , normal baryon octet  $B$  and meson octet  $M$  is

$$\begin{aligned} \mathcal{L}'_1 &= G_1 \bar{\Lambda}_1 B_j^i M_i^j + H.c. \\ &= G_1 \bar{\Lambda}_1 (K_c N + \pi \Sigma + K \Xi + \eta_0 \Lambda) + H.c., \end{aligned} \quad (16)$$

where

$$\pi = (\pi^-, \pi^0, \pi^+), \quad K = (K^0, K^+), \quad K_c = (K^-, \bar{K}^0). \quad (17)$$

Since the parity of these pentaquarks is negative, they will decay via S-wave if kinematics allows. With the mass values in Table II, it is easy to find out which decay process will occur.

According to our mass estimate, only  $p_8, n_8$  are below the threshold of the listed decay modes in Table IV. At first sight, there is no strong decay modes for them. They should be stable particles.

However, there are multiple-pion decays modes which violate the "fall-apart" mechanism, such as S-wave  $p_8 \rightarrow N\pi$ , P-wave  $p_8 \rightarrow N\pi\pi$  and S-wave  $p_8 \rightarrow N\pi\pi\pi$  where  $N$  is either a proton or neutron.

Another possibility is the isospin violating strong decay mode  $p_8 \rightarrow p\eta_0 \rightarrow p\pi$ . The virtual intermediate state  $p\eta_0$  helps this process happen. The first step satisfies the "fall-apart" mechanism. Then the virtual  $\eta_0$  turns into a real pion through isospin violating effects. All these processes contribute to the decay width of  $p_8, n_8$ . However both  $p_8$  and  $n_8$  should still be narrow resonances.

#### IV. ADDITIONAL SELECTION RULES IN THE IDEAL MIXING CASE FOR THE $I = 0$ SECTOR

For the  $I = 0$  channel, physical states are the mixture of octet and singlet states. For example, the physical  $\eta, \eta'$  are the mixture of  $\eta_0$  and  $\eta_1$  where  $\eta_0$  is the pure octet member and  $\eta_1$  is the pure singlet. In the following we will let the mixing angle deviate from the physical value.

$$\begin{aligned} \eta &= \eta_0 \cos \theta - \eta_1 \sin \theta \\ \eta' &= \eta_0 \sin \theta + \eta_1 \cos \theta. \end{aligned} \quad (18)$$

From the above we have

$$\begin{aligned} \eta_0 &= \eta' \sin \theta + \eta \cos \theta \\ \eta_1 &= \eta' \cos \theta - \eta \sin \theta. \end{aligned} \quad (19)$$

The mixing of  $\Lambda_8$  and  $\Lambda_1$  is defined as

$$\begin{aligned}\Lambda_n &= \Lambda_8 \cos \varphi - \Lambda_1 \sin \varphi \\ \Lambda_s &= \Lambda_8 \sin \varphi + \Lambda_1 \cos \varphi.\end{aligned}\quad (20)$$

Now the interaction terms involving  $I = 0$  states are

$$\begin{aligned}\mathcal{L}_{mixing} &= g_8 \{ (\bar{\Sigma}_8 \Sigma + \bar{\Xi}_8 \Xi) \eta' \left( \frac{1}{\sqrt{6}} \sin \theta + a \cos \theta \right) \\ &+ (\bar{\Sigma}_8 \Sigma + \bar{\Xi}_8 \Xi) \eta \left( \frac{1}{\sqrt{6}} \cos \theta - a \sin \theta \right) \\ &+ \bar{N}_8 N \eta' \left( -\frac{2}{\sqrt{6}} \sin \theta + a \cos \theta \right) \\ &+ \bar{N}_8 N \eta \left( -\frac{2}{\sqrt{6}} \cos \theta - a \sin \theta \right) \\ &+ \bar{\Lambda}_s (\pi \Sigma + K_c N) \left( \frac{1}{\sqrt{6}} \sin \varphi + b \cos \varphi \right) \\ &+ \bar{\Lambda}_n (\pi \Sigma + K_c N) \left( \frac{1}{\sqrt{6}} \cos \varphi - b \sin \varphi \right) \\ &+ \bar{\Lambda}_s K \Xi \left( -\frac{2}{\sqrt{6}} \sin \varphi + b \cos \varphi \right) \\ &+ \bar{\Lambda}_n K \Xi \left( -\frac{2}{\sqrt{6}} \cos \varphi - b \sin \varphi \right) \\ &+ \bar{\Lambda}_s \Lambda \eta' \left( -\frac{1}{\sqrt{6}} \sin \varphi \sin \theta + a \sin \varphi \cos \theta + b \cos \varphi \sin \theta \right) \\ &+ \bar{\Lambda}_s \Lambda \eta \left( -\frac{1}{\sqrt{6}} \sin \varphi \cos \theta - a \sin \varphi \sin \theta + b \cos \varphi \cos \theta \right) \\ &+ \bar{\Lambda}_n \Lambda \eta' \left( -\frac{1}{\sqrt{6}} \cos \varphi \sin \theta + a \cos \varphi \cos \theta - b \sin \varphi \sin \theta \right) \\ &+ \bar{\Lambda}_n \Lambda \eta \left( -\frac{1}{\sqrt{6}} \cos \varphi \cos \theta - a \cos \varphi \sin \theta - b \sin \varphi \cos \theta \right) \} \\ &+ H.c. .\end{aligned}\quad (21)$$

where  $a = \frac{g_1}{g_8}, b = \frac{G_1}{g_8}$ .

In the extreme case of ideal mixing, i.e.,  $\tan \theta = \tan \varphi = -\sqrt{2}$ , we have

$$\begin{aligned}\Lambda_s &= [su][ds]_{-\bar{s}} \\ \Lambda_n &= \frac{1}{\sqrt{2}} ([ud][su]_{-\bar{u}} + [ds][ud]_{-\bar{d}}) \\ \eta' &= s\bar{s} \\ \eta &= \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}).\end{aligned}\quad (22)$$

The so-called "fall-apart" mechanism requires that there is no annihilation or creation of quark pairs when pentaquarks decay. Hence the coefficient of the fourth and eighth terms must vanish in the limit of ideal mixing. In this way, we get

$$a = b = \frac{1}{\sqrt{3}}. \quad (23)$$

The three coupling constants  $g_1, G_1, g_8$  are related to each other in this limit.

Now Eq. (21) has a simple form

$$\begin{aligned}\mathcal{L}_{mixing} &= \frac{1}{\sqrt{2}} (\bar{\Sigma}_8 \Sigma + \bar{\Xi}_8 \Xi) \eta + \bar{N}_8 N \eta' \\ &+ \frac{1}{\sqrt{2}} \bar{\Lambda}_n (\pi \Sigma + K_c N) + \bar{\Lambda}_s K \Xi \\ &- \frac{2}{\sqrt{6}} \bar{\Lambda}_s \Lambda \eta' + \frac{1}{\sqrt{6}} \bar{\Lambda}_n \Lambda \eta + (H.c.).\end{aligned}\quad (24)$$

The decay modes of  $\Lambda_s$  and  $\Lambda_n$  are

$$\Lambda_s \longrightarrow K \Xi - \frac{2}{\sqrt{6}} \Lambda \eta' \quad (25)$$

$$\Lambda_n \longrightarrow \frac{1}{\sqrt{2}} (\pi \Sigma + K_c N) + \frac{1}{\sqrt{6}} \Lambda \eta. \quad (26)$$

The above relation is the selection rule from the "fall-apart" mechanism in the ideal mixing limit.

It's very interesting to note that the only dynamically allowed two decay modes of  $\Lambda_s$  are kinematically forbidden since  $\Lambda_s$  is below the threshold. Therefore,  $\Lambda_s$  will not decay via strong interaction. It is a long-lived stable particle in the ideal mixing case. For  $\Lambda_n$  the only both dynamically and kinematically allowed decay mode is  $\pi \Sigma$ . Unfortunately, the physical  $\eta$  and  $\eta'$  are not ideally mixed. So the results obtained in this section may be different from the realistic case, therefore not very useful.

## V. DISCUSSION

We have shown that there exist an octet and singlet pentaquark multiplet with  $J^P = \frac{1}{2}^-$  in the framework of Jaffe and Wilczek's diquark model. We have calculated their masses and magnetic moments. Several interesting mass and magnetic moment relations are derived. We have also constructed the chiral Lagrangian for these pentaquarks. Possible strong decay modes are discussed. We have derived selection rules based on the "fall-apart" decay mechanism. In this limit there exists a long-lived stable  $J^P = \frac{1}{2}^-$   $\Lambda_s$  pentaquark which will not decay via strong interaction.

Because there is no orbital excitation within these nine pentaquarks, their masses are lower than the anti-decuplet and the accompanying octet with positive parity. Their masses range between 1360 MeV and 1540 MeV according to our calculation using the same mass formula in [25]. These states are close to the L=1 orbital excitations of the nucleon octet. The mixing between the pentaquark states and orbital excitations is expected to be small since their spatial wave functions are very different.

According to our calculation, two of the  $J^P = \frac{1}{2}^-$  octet pentaquark members  $p_8, n_8$  lie 22 MeV below the  $p\eta_0$  threshold and 228 MeV below the  $\Sigma K$  threshold. The "fall-apart" decay mechanism forbids  $p_8$  to decay into one nucleon and one pion. Although their interaction is

of S-wave, lack of phase space forbids the strong decays  $p_8 \rightarrow p\eta_0, \Sigma K$  to happen.

For  $p_8$ , the only kinematically strong decays are S-wave  $p_8 \rightarrow p\pi$ , P-wave  $p_8 \rightarrow N\pi\pi$  and S-wave  $p_8 \rightarrow N\pi\pi\pi$  decays where  $N$  is either a proton or neutron. All these decay modes involve the annihilation of a strange quark pair and violate the "fall-apart" mechanism. Hence the width is expected to be small. Both  $p_8$  and  $n_8$  may be narrow resonances.

It is interesting to note there are three negative-parity  $\Lambda$  particles within the range between 1400 MeV and 1540 MeV if Jaffe and Wilczek's diquark model is correct. One of them is the well established  $\Lambda(1405)$ .  $\Lambda(1405)$  is only 30 MeV below kaon and nucleon threshold. Some people postulated it to be a kaon nucleon molecule [48]. We propose that there is another intriguing possibility of interpreting  $\Lambda(1405)$  as the candidate of  $J^P = \frac{1}{2}^-$  pentaquark. The other  $J^P = \frac{1}{2}^-$  pentaquark and the

corresponding  $L=1$  singlet  $\Lambda$  particle may have escaped detection so far.

The discovery of nine additional negative-parity baryons in this mass range will be strong evidence supporting the diquark model. On the other hand, if future experimental searches fail to find any evidence of these additional states with negative parity, one has to re-evaluate the relevance of the diquark picture for the pentaquarks.

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